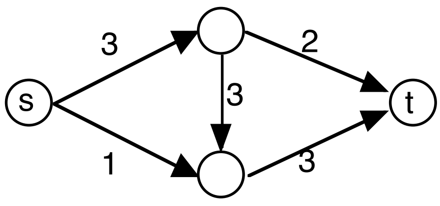
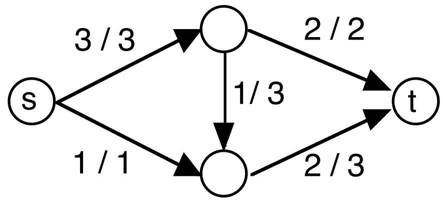
Flow in transport networks



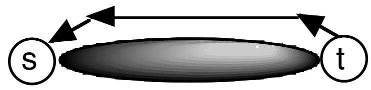
s: source, start
t: sink, terminate
weights >0: arc capacity
f / w: flow and capacity



Df. transport network:
G = (V, E, w, s, t), $E \subset V \times V$, e = u-v with $u \neq v$ (no loops), w: E $\rightarrow$ **R+** (positive reals), $s, t \in V$, $s \neq t$

Df. flow f : E $\rightarrow$ **R+** =[0 .. ∞)
a) obey capacity: for all $e \subset E$, $0 \leq f(e) \leq c(e)$
b) conservation of flow (Kirchhoff law): for all $v \in V$ except s and t: in-flow = out-flow

Introduce a feedback arc t-s of capacity w(t-s) = ∞
Df. value | f | of flow f: | f | = f(t-s)

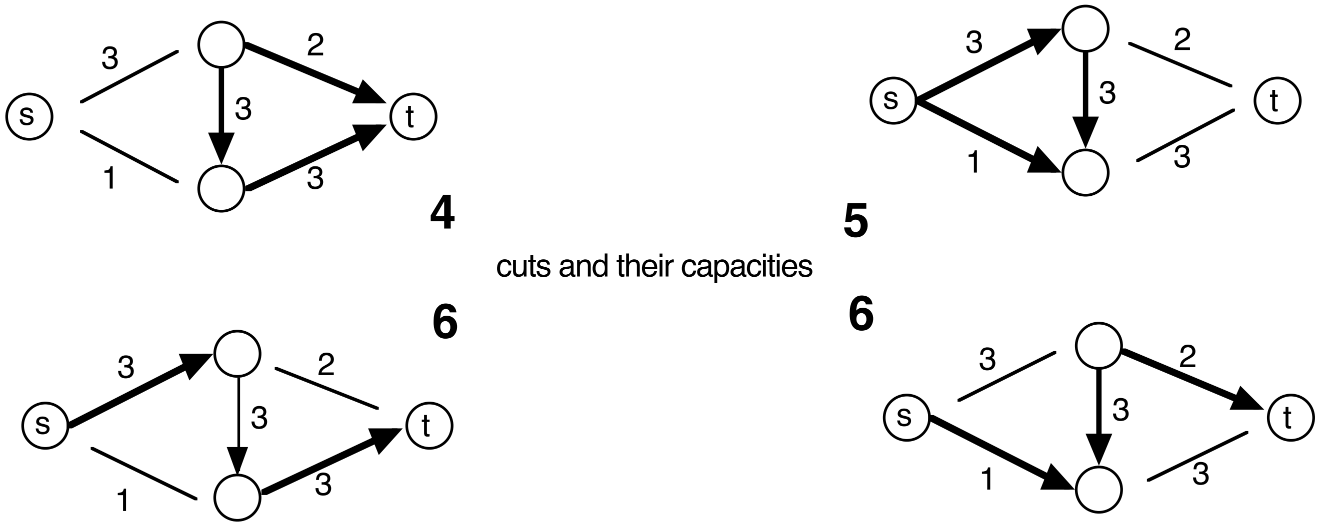


Problem: construct a flow f from s to t of maximum value | f |

Linear programming, linear optimization: maximize $f = \Sigma_u\ f(s\text{-}u) = \Sigma_u\ f(u\text{-}t)$ subject to $0 \leq f(e) \leq c(e)$

Df. cut (S, T) = a partition of V: $S \cap T = \{ \}$, $S \cup T = V$, $s \in S$, $t \in T$

Intuition: cut as a set of edges: cut (S, T) = { e = u-v / $u \in S$, $v \in T$ }



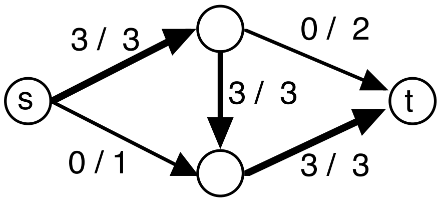
cuts and their capacities

Df. capacity w(S, T) of cut (S, T) = Σ w(u-v), summed over all $u \in S$, $v \in T$

Df. flow f(S, T) across cut(S, T) = Σ f(u-v) - Σ f(v-u), summed over all $u \in S$, $v \in T$

Max-flow min-cut theorem (Ford & Fulkerson 1956):
In a transportation network the maximum value | f | over all flows f equals the minimum value w(S, T) over all cuts(S, T)

The concept of an augmenting path.



the flow at left leaves the augmenting path shown at right of capacity 1